Trading on Sunspots

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Abstract

In a model with multiple Pareto-ranked equilibria we endogenize the equilibrium selection probabilities by adding trade in assets that pay based on the realization of a sunspot. Asset trading imposes restrictions on the equilibrium set. When the probability of a low outcome is high enough, the coordination game becomes more like a prisoner's dilemma in which the high equilibrium disappears because of the asset positions that agents trade towards induce some agents to withhold their effort. We derive an upper bound on the probabilities of the low-level equilibrium that we interpret as a disaster. We derive asset pricing implications including the disaster premium, and we study the effect of shocks to beliefs over actions and the implied news in stock prices.

1 Introduction

In a coordination game with multiple Pareto-ranked equilibria, an equilibrium can be chosen by an extrinsic device such as a sunspot. The mapping between sunspots and equilibria is in most of not all models exogenous, as is the distribution of the sunspot and, hence, the distribution of equilibria. The sunspot is a public signal that correlates players' actions.

We show that, roughly speaking, the probability of the low equilibrium is high enough (but less than unity), trading on the equilibrium-choice signal – the sunspot – may transform the coordination game into a prisoner's dilemma game with a unique low-level equilibrium. Of course, for trading on sunspots to take place at all, their impact on equilibria must be non-degenerate.

Our paper distinguishes between sunspots and the equilibria that result therefrom. The mapping from sunspots to equilibrium play is endogenous. This is done through a prior stage in which, before taking a real action, players trade securities that pay contingent on the realization of a sunspot. The choice of which securities to trade reflects the Nash equilibrium beliefs that determine what sunspot state maps into what aggregate action profile and, hence, the probabilities with which equilibria arise, as well as the set of equilibria.

Adding a prior stage generally changes equilibrium play in the subsequent game itself. In our context, the prior stage entails trading on sunspots, and it bounds from above the probability with which the "low" (also referred to as "disaster") equilibrium is chosen. The set of equilibrium distributions therefore shrinks.

In general, asset trade can destroy equilibria, or create new ones, or leave the equilibrium set unchanged. We argue that we have uncovered an intuitively appealing mechanism whereby asset trading destroy the high equilibrium. The model has two types of agents, rich and poor. Each type is better off in the high equilibrium than in the low equilibrium, but when equilibria are chosen by a sunspot, both types face aggregate risk. This risk cannot be eliminated, but the consumption of the two types can be made more correlated if they trade on the sunspot's realization Poor agents especially wish to insure against the low equilibrium, but this means that they must pay the rich in the high equilibrium. But the rich would in that event become even richer and, because their utility of consumption is concave, they may not be willing to exert the effort they would exert if they did not purchase any assets. If the transfer of resources is large enough, the high equilibrium fails to exist.

We stress the similarity to the prisoner's dilemma because allowing for asset trade reduces all agents' expected utility. The type that wants to deviate from the high equilibrium is the rich type. He finds it optimal to buy claims to so much income in the high state that he will not want to work. But the existence of the high equilibrium requires that both types exert effort.

How is the disaster probability endogenized? The size of the transfer between the rich and the poor rises with the probability that the low equilibrium will occur. For the low equilibrium to exist, its probability must be low enough. So, that the transfer to the rich in high states is low enough that the high equilibrium can exist. Agents trade portfolios of Arrow securities. A continuum of such securities exists. The composition of a portfolio determines the probability that it pays. Asset trade at the first stage changes agents' payoffs at the production stage. This then determines the equilibrium set at the production stage. A high probability of disasters cannot arise, for it would create too large a transfer to the rich in those sunspot states that are believed to lead to the high equilibrium. Such probabilities are therefore not consistent with equilibrium, and this is the sense in which equilibrium selection is endogenized. Generally, the set of surviving disaster probabilities has an open interior – we cannot, in other words, derive the probability uniquely

Our paper adds to several lines of research:

(*i*) The correlated equilibrium concept of Aumann (1974) in which mediator sends out a vector of messages, one message to each player. The distribution of the message vector is common knowledge among the players and each player maximizes his utility conditional on the message that he has received. The set of correlated equilibria depends on the structure of the game, and our paper provides an example in which the predictive content of the game is raised by adding a prior asset-trading stage.

(*ii*) The research on trade on sunspots; Peck and Shell (1991) and Forges and Peck (1995) take the probabilities of equilibrium selection to be exogenous; we endogenize these probabilities.

(*iii*) The research on news shocks, in particular shocks to beliefs about the actions of others, such as studied by Angeletos and La'O (2014). In contrast to them, we have multiple equilibria. We replicate Hall's (1988) finding that consumption reacts to lagged consumption and the stock-price index.

(*iv*) The research on coordination failures as causes of the real business cycle; Benhabib and Farmer (1999), and to asset pricing – Lagos and Zhang (2013) and Benhabib and Wang (2014). It does not study how asset trading may restrict equilibrium actions and their probabilities.

(*v*) The research on coordination failures as causes of bank runs; based on logic different from ours, Pauzner and Goldstein (2005) derive a unique probability of a bank-run in the model of Diamond and Dybvig (1983). Our method generally leads us only to a range of admissible disaster probabilities.

(*vi*) The literature on disasters and their relation to asset pricing. We focus on coordination failures, but there are other disasters such as wars and natural catastrophes. Our asset prices display a disaster premium that is related to disaster size and its probability. The disaster size is 0.29 and the disaster probability is only 2% both are similar to the estimates in Barro (2006). As the probability of disasters increases, the premium grows. However, a higher than 2% probability of a disaster is not sustainable as trading in the financial markets changes the set of possible equilibria. Thus, we provide a theory of disaster risk. Additionally, we find that in probable contrast to wars and natural catastrophes, the size of disasters and their frequency are positively correlated across equilibria: The larger the disaster, the higher is the likelihood that it can occur.

(*vii*) Research on how financial development relates to real activity. Financial markets reduce the incidence of disaster outcomes and they reduce the inequality of consumption in disasters; in this sense they are beneficial.

Plan of paper.—We begin with a model without capital markets. We then show how capital markets restrict the equilibrium set. We then look at asset pricing, the disaster premium and the effects of news shocks as manifested through changes in asset prices. Finally we study how news shocks about the actions of others manifest themselves in stock prices, and the induced correlation between stock prices and real activity.

2 The model

Consider a production economy with two types of individuals lasting one period.

Endowments.—Type i receives endowment z_i, with $0 < z_1 < z_2$. The fraction of type i agents is f_i.

Preferences.—Utility depends on consumption $c \geq 0$ and effort $x \in \{0, 1\}$:

$$U(c) - \kappa x, \tag{1}$$

where κ is the disutility of effort.

Production.—Let

$$\bar{x} = \sum f_i x_i$$

denote the per-capita effort. We restrict our attention to symmetric pure strategy equilibria in which all agents of one type exert the same effort. As a function of own effort x and aggregate effort $\bar{x}$, an agent's output is

$$y(x, \bar{x}) = (\alpha + \bar{x})x.$$

Aggregate output is zero when $\bar{x} = 0$, and $1 + \alpha$ when $\bar{x} = 1$.

Consumption.—Consumption takes place after production has taken place and after assets and obligations are settled. If financial markets are closed, an agent consumes his endowment z and his output y which are his only sources of income. That is, $c = z + y(x, \bar{x}) > 0$. If financial markets are open, consumption also includes asset payoffs.

Disaster size.—Aggregate consumption in the low equilibrium relative to that in the high equilibrium is

$$\frac{\bar{z}}{\bar{z} + 1 + \alpha} \geqslant 0.71, \tag{2}$$

where $\bar{z} = f_1 z_1 + f_2 z_2$ is the average endowment. The lower bound in (2) is based on estimated from Barro (2006).

Aggregate shocks.—The model has no intrinsic shocks. There is an extrinsic variable called a "sunspot." We depart from the literature in that we have more sunspot realizations than there are equilibria. In fact, the sunspot can take on a continuum of values, as does temperature for example. The distribution of the sunspot variable is exogenous, but the mapping between sunspots and equilibrium play is endogenous.

The mechanism that endogenizes the mapping is agents' selection of what portfolio to trade, and the resulting beliefs concerning equilibrium play. As the agents choose which portfolio they want to trade among themselves, they will endogenize the probabilities with which the equilibria are selected. That is, they will endogenize the mapping between the sunspots and equilibrium play. We now define our terms more precisely.

Sunspots.—A sunspot is an exogenous random variable s that is uniformly distributed on $[0, 1]$ or, more formally, has Lebesgue measure $\mu(s)$ over the Borel subsets of $[0, 1]$. When financial markets are open, securities pay as a function of s. We start with the setting in which there are no financial markets.

The space of sunspot realizations is rich enough that it can be transformed into any other space of realizations. One can generate two conceptually different types of financial markets. One is for securities that pay depending on some other extrinsic random variable taking on values in some set other than $[0, 1]$. But, this can be shown to be equivalent to trading assets contingent on realizations in $[0,1]$; one simply needs to change the probabilities associated with the new set of realizations.

Another market type, more relevant empirically, is for securities that pay based on outcomes that depend, at least in part, on actions that agents take, outcomes such as aggregate output. Our methods apply to such cases as well, as we explain in Section 3.

2.1 Equilibrium without financial markets

When financial markets are closed effort, x, is the only action. An agent's action can depend on his endowment, z, and on the sunspot, s. When the equilibrium is symmetric an agent's strategy is a function $x : \{z_1, z_2\} \times [0,1] \to \{0,1\}$.

Nash Equilibrium with no assets.—A Nash equilibrium is a function x such that for all $(z,s) \in \{z_1, z_2\} \times [0,1]$,

$$x(z,s) \in \arg\max_{x \in \{0,1\}} \{U(z + y[x, \bar{x}(s)]) - \kappa x\} \tag{3}$$

where

$$\bar{x}(s) = \sum_{i=1}^{2} f_i x(z_i, s) \tag{4}$$

the following equilibria may arise at a particular sunspot realization s:

Equilibrium "L".—In the first type of equilibrium $x(z,s) = \bar{x}(s) = 0$ for all z. No individual works. We call this a "low" equilibrium, or equilibrium L. For this to be an equilibrium we need the following two conditions:

$$U(z_1) \geqslant U(z_1 + \alpha) - \kappa,$$
$$U(z_2) \geqslant U(z_2 + \alpha) - \kappa.$$

That is, if $\bar{x}$ is zero, the reward to working is just α, and each type should prefer not to work. Because U is concave it is sufficient that the poor are *not* willing to work:

$$U(z_1 + \alpha) - U(z_1) \leqslant \kappa. \tag{5}$$

Equilibrium "H".—At the other extreme, everyone exerts effort and $x\,(z, s) = \bar{x}\,(s) = 1$. We call this a "high" equilibrium, or equilibrium H. For this equilibrium to exist we need the following two conditions:

$$U(z_1 + \alpha + 1) - \kappa \geq U(z_1),$$
$$U(z_2 + \alpha + 1) - \kappa \geq U(z_2).$$

Again, because U is concave it is sufficient that the rich *are* willing to work:

$$U(z_2 + \alpha + 1) - U(z_2) \geqslant \kappa. \tag{6}$$

In equilibria H and L, every agent takes the same action – either every agent exerts effort or no agent does. There generally are, however, other equilibria and some of these are symmetric pure strategy equilibria, some not. In all these equilibria some agents exert effort while others do not.

Equilibrium M.—In this equilibrium only the poor exert effort and $\bar{x} = f_1$. We call this a "middle" equilibrium, or equilibrium M. For this equilibrium to exist we need the following two conditions:

$$U(z_1 + \alpha + f_1) - \kappa \geqslant U(z_1),$$
$$U(z_2 + \alpha + f_1) - \kappa < U(z_2).$$

Neither condition implies the other. This is also a symmetric equilibrium. Note that the conditions guaranteeing equilibria H and L do not involve the f_i, the conditions involving the existence of equilibrium M do depend on the f_i.

Asymmetric equilibria.—In these types of games the number of equilibria is generically odd. This means that when L and H both exist (see Proposition 1), there will also be a third equilibrium. This third equilibrium will either by asymmetric so that a fraction of agents of some type play $x = 1$ while the remainder play $x = 0$, or it will be the symmetric equilibrium M.

We shall assume that equilibrium M and the asymmetric equilibria are never chosen. If they sometimes were chosen, the number of cases proliferates, but nothing conceptually new is added. Thus we only admit L and H as possibilities. Sometimes only L exists, sometimes only H, and sometimes both H and L do.

Next, we define the parameter set under which both H and L exist, which is the set of parameters for which (5) and (6) both hold:

Definition 1. *Let* $\mathcal{P}_{aut} = \{(z_1, z_2, \kappa) : U(z_2 + \alpha + 1) - U(z_2) \geqslant \kappa \geqslant U(z_1 + \alpha) - U(z_1)\}$.

Roughly speaking, if α is high relative to κ, equilibrium L does not exist, and if α is low relative to κ, equilibrium H does not exist. If neither extreme obtains, L and H both exist. The set $\mathcal{P}_{aut}$ is always non-empty. To see this fix α. Then for any $z > 0$ we have $(\kappa, z_1, z_2) = (U(z+\alpha+0.5)-U(z), z, z) \in \mathcal{P}_{aut}$. That is there is a set, with a non-empty interior, where both the low and the high equilibria exist. Intuitively, endowment z_1 must not be too low as then type-1 individuals would always work and the L equilibrium would not exist. Endowment z_2 must not be too high as then type-2 individuals would never work and the H equilibrium would not exist.

Proposition 1. *Let* $U(c) = \ln c$. *Then there exists a non-empty set of parameters* $(z_1, z_2, \alpha, \kappa)$ *such that equilibria L and H both exist.*

Let $\delta = 1/(e^{\kappa}-1) \approx 1/\kappa$. Then in a special case with logarithmic preferences we have:

$$\mathcal{P}_{ut} = \{(z_1, z_2, \alpha, \delta) : \alpha\delta \leqslant z_1 \leqslant z_2 \leqslant (\alpha+1)\delta\}. \tag{7}$$

Figure 1 summarizes our findings. Region L(H) denotes the set of endowments for which only the L (H) equilibrium exists. Our main interest is in region H+L that consists of endowments such that both the L and the H equilibria exist. In what follows we study conditions under which this set persists when allow individuals to trade financial securities contingent on sunspots and the sunspots will be correlated with the type of equilibrium that is played at the production stage. The unmarked top left corner is where neither of the two equilibria exists.[1]

2.1.1 Equilibrium selection without financial markets

Let $\mathbf{L} \subset [0, 1]$ be the set of s realizations that lead to equilibrium L, and $\mathbf{H} = [0, 1] \setminus \mathbf{L}$ the set of s realizations that lead to equilibrium H. Define the

[1]In this region there exist equilibria with $\bar{x} \in (0, 1)$. In such equilibria a fraction of individuals of the type 1 invests while others do not.

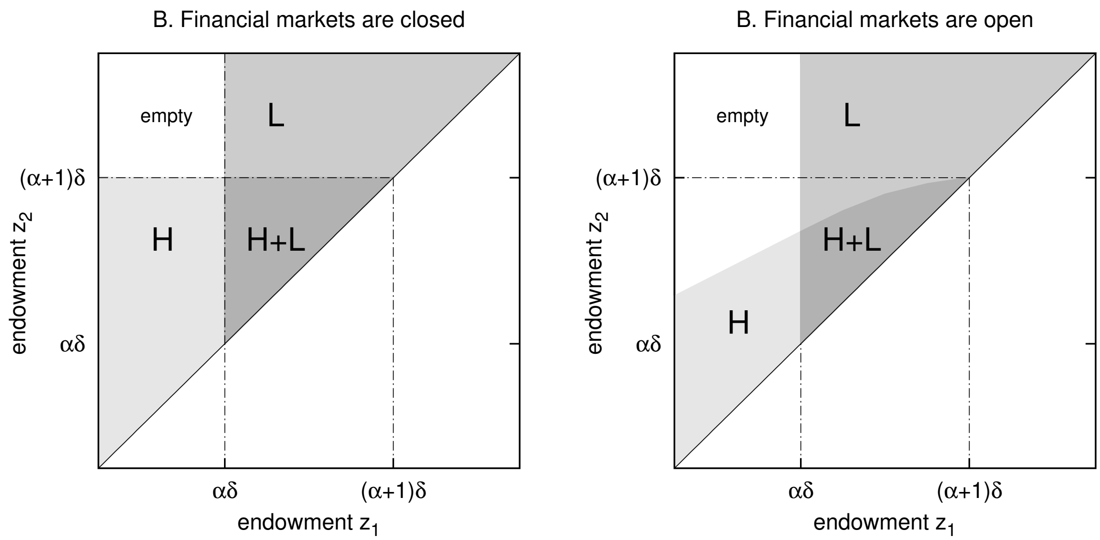

Figure 1: Equilibrium map

equilibrium indicator $\omega(s) \in \{\mathrm{L}, \mathrm{H}\}$ as follows:

$$\omega = \begin{cases} L & \text{if} \quad s \in \mathbf{L} \\ H & \text{if} \quad s \in \mathbf{H} \end{cases} \tag{8}$$

Thus the probabilities of the two equilibria being played are

$$\Pr(\omega = L) = \pi^L \quad \text{and} \tag{9}$$

$$\Pr(\omega = H) = \pi^{\mathrm{H}} \ , \tag{10}$$

where

$$\pi^L \equiv \mu(\mathbf{L}) \quad \text{and} \quad \pi^{\mathrm{H}} \equiv \mu(\mathbf{H}) = 1 - \pi^L. \tag{11}$$

Any pair $\left(\pi^{\mathrm{H}}, \pi^L\right)$ of non-negative numbers summing to unity is admissible when there are no assets. With assets in the model, however, that is no longer true.

2.2 Equilibrium with financial markets

Arrow securities.—An Arrow security is in zero net supply, and pays a unit of consumption in a particular sunspot state s, and zero otherwise, and its price is $Q(s)$. There is a continuum of such securities, one for each s. Now an agent of type z has an additional set of actions consisting of the number of securities, $N(z,s)$ to hold as claims to consumption in state s. This adds for each agent a trading strategy $N : \{z_1, z_2\} \times [0,1] \to R$. Market clearing then requires that for each $s \in [0,1]$

$$\sum_{i=1}^{2} f_i N(z_i, s) = 0 \tag{12}$$

Budget constraint.—$N(z,\cdot)$ is agent z's *portfolio*. An agent trades before he receives his endowments and before he receives the output that he will have produced with the effort that he has expended. His endowment is not contractible and his trades must therefore net out to zero. For a type-z agent, the portfolio $N(z,\cdot)$ must then satisfy[2]

$$\int_0^1 Q(s)\, N(z,s)\, d\mu(s) = 0. \tag{13}$$

Nash Equilibrium with asset trade.—It consists of three functions, $Q : [0,1] \to R_{++}$, and $(x, N) : \{z_1, z_2\} \times [0,1] \to \{0,1\} \times R$ such that (12) holds and such that for all $(z,s) \in \{z_1, z_2\} \times [0,1]$,

$$N(z,s) = \arg\max_{N(\cdot)} \int_0^1 \max_{x \in \{0,1\}} \left[U\left(z + y\left[x, \bar{x}(s)\right] + N(z,s)\right) - \kappa x \right] d\mu(s) \tag{14}$$

subject to (13), and such that

$$x(z,s) = \arg\max_{x \in \{0,1\}} \left\{ U\left(z + y\left[x, \bar{x}(s)\right] + N(z,s)\right) - \kappa x \right\}, \tag{15}$$

[2]It must be true that $N(z,s) = c(z,s) - z - y(s)$ for each type. The corresponding "inter-temporal" budget constraint then is:

$$\int_0^1 Q(s) c(z,s) ds = z + \int_0^1 Q(s) y(s) ds.$$

Our first stage budget constraint does not include z. The alternative budget constraint formulation is: $\int_0^1 Q(s)\, N(z,s)\, ds = z$. It implies the same intertemporal budget constraint, because then $N(z,s) = c(z,s) - y(s)$, and, so, leaves the solution unchanged. This would not be true if the individuals had to make their portfolio decisions before they knew their type z. In this case there would be incentives to insure against the risk of being a type 1.

where $\bar{x}(s)$ is given in (4). Strictly speaking there are 4 functions, $(Q(s), \bar{x}(s), x(z,s), N(z,s))$ satisfying (4), (12), (14) and (15).

Simple portfolios.—A simple portfolio of Arrow securities is an allocation that places equal weights on all those securities in which an agent is long and equal weights on those in which he is short. That is, for any subset $A \subseteq [0,1]$

$$N(z,s) = \begin{cases} N_A & \text{for } s \in A \\ N_{\sim A} & \text{for } s \in {\sim}A \end{cases}.$$

A simple portfolio places equal weights on the securities $s \in A$, and an equal weight on securities with $s \in {\sim}A$, so that (13) reads

$$N_A \int_A Q^s d\mu(s) = -N_{\sim A} \int_{\sim A} Q^s d\mu(s). \tag{16}$$

Other, unequally-weighted bundles are also possible, but deviations to such portfolios will not raise any agent's utility as we shall show later. We shall adopt the convention that $N_A \geq 0$ and $N_{\sim A} \leq 0$, i.e., we shall label A for the set of securities that are assets in portfolio A, with the remainder being liabilities. Then a portfolio is characterized fully by two numbers: (A, N_A). Given this pair we then infer $N_{\sim A}$ from the budget constraint. Therefore, we shall refer to a portfolio as "portfolio A." An agent can trade portfolio A at any scale indexed by N_{A}.

Portfolio payoffs.—Let $w(A)$ denote the payoff of portfolio A. Then

$$w(A) = \begin{cases} N_A & \textit{if}\, s \in A \\ N_{\sim A} & \textit{if}\, s \in {\sim}A \end{cases}. \tag{17}$$

Probability of a positive payoff for portfolio A is denoted by π^A:

$$\pi^A \equiv \mu(A). \tag{18}$$

Portfolio choices.—This choice is made after the agents have discovered the z_i that they will be receiving prior to consumption. There are only two types of agents indexed by their endowments, rich and poor, only two portfolios will be chosen in equilibrium. Let A denote the portfolio chosen by the poor. The rich will take the other side of each s-security trade, and so the rich choose portfolio ${\sim}A$.

This equilibrium selection is consistent with trades in that the poor wish to receive income if equilibrium L arises, and they pay the rich if equilibrium H arises which occurs because the preferences we assume have the property that $U''' > 0$.

Figure 2 illustrates a portfolio of a poor agent who is long on securities $s \in A$, and short on securities $s \in \tilde{} A$.

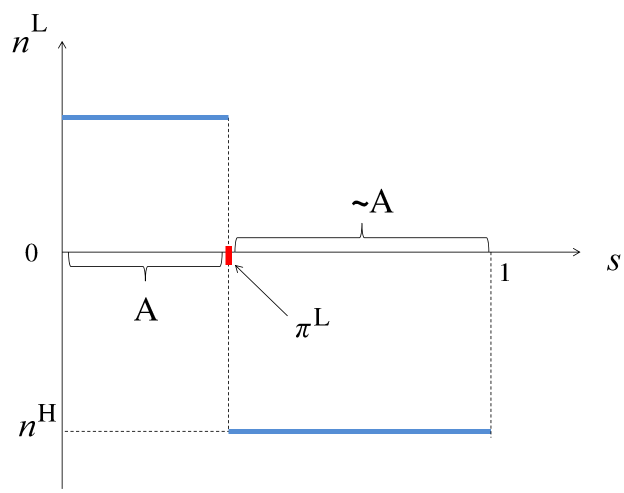

Figure 2: Portfolio of a low-endowment individual (z_1)

Trading strategies as functions of belief formation over $\bar{x}$.—Nash equilibrium beliefs are over the profile of others' actions in state s. In particular, the profile in question is the function $x(z, s)$. An agent cares only about the per-capita action of others, $\bar{x}(s)$, which is the following function of the sunspot:

$$\bar{x}(s) = \begin{cases} 0 & \text{if } s \in \mathbf{L} \Leftrightarrow \ \omega = \ \text{L} \\ 1 & \text{if } s \in \mathbf{H} \Leftrightarrow \ \omega = \ \text{H} \end{cases},$$

The financial-markets-open game *de facto* introduces just two additional actions, namely

(i) which portfolio A to trade, and

(ii) what quantity N_{A} to trade.

c) Sufficiency requires that neither agent type wants to deviate to a different portfolio, i.e., to a set $A \neq \mathbf{L}$. What the agent wants is insurance.

Given the beliefs specified above, however, his production income depends on ω alone. At the equilibrium portfolio, the same is true for his asset income. In other words, for the poor agent, asset income is perfectly negatively correlated with his production income, whereas for the rich, asset income is perfectly positively correlated. We show that because $U''' > 0$, the poor are priced out of claims in states $s \in H$ and the rich are priced out of claims in state $s \in L$.

Trading equilibrium.—An equilibrium entails simple portfolios for all agents. They are of the form

$$\begin{aligned} A = \mathbf{L} \quad &\text{and} \quad \sim A = \mathbf{H} \quad \text{for the poor,} \\ A = \mathbf{H} \quad &\text{and} \quad \sim A = \mathbf{L} \quad \text{for the rich.} \end{aligned} \tag{19}$$

That is, the disaster states $s \in \mathbf{L}$ entail transfers to the poor, whereas states $s \in \mathbf{H}$ entail transfers to the rich.

Once A is given, all securities $s \in \mathbf{L}$ will have the same price that we shall denote by Q^{L}, and all securities $s \in \mathbf{H}$ will have the same price that we shall denote by Q^{H}. Then

$$q^{\mathrm{L}} = \pi^{L} Q^{\mathrm{L}} \quad \text{and} \quad q^{\mathrm{H}} = \pi^{\mathrm{H}} Q^{\mathrm{H}}.$$

For the equal-weighted assets and equal-weighted liabilities portfolios we shall now use the notation

$$N(z,s) \equiv \begin{cases} n_z^{\mathrm{L}} \ \mathit{if}\, s \in \mathbf{L} \\ n_z^{\mathrm{H}} \ \mathit{if}\, s \in \mathbf{H} \end{cases}.$$

In that case these new definitions and (16) imply that type-i agents' asset trades must satisfy the following budget constraint

$$q^{L} n_z^{\mathrm{L}} + q^{H} n_z^{\mathrm{H}} = 0.$$

We assume that the portfolio choices are made simultaneously and non-cooperatively. Each security trades at the price q^{L} if $s \in \mathbf{L}$ or q^{H} if $s \in \mathbf{H}$. A trading equilibrium is then indexed by $\mathbf{L}$, and associated with these equilibria is a "disaster probability" π^{L}, defined in (18). Not all $\pi^{L} \in [0,1]$ are equilibria, as we shall see, but generally a continuum exists.

Further suppose that there operate financial markets that trade portfolios paying one unit of consumption good conditional on the realization of ω.

Security L (H) pays one unit if and only if state $\omega = L$ ($\omega = H$) realizes. Security ω is traded at price q^ω and the trade occurs before endowments are delivered. We let n_z^ω to denote quantity of securities ω purchased by a type-z individual. An individual of type $z \in \{1, 2\}$ faces the following budget constraint:

$$q^L n_z^L + q^H n_z^H = 0. \tag{20}$$

Financial market clearing conditions for securities L and H are:

$$f_1 n_1^L + f_2 n_2^L = 0, \tag{21a}$$

$$f_1 n_1^H + f_2 n_2^H = 0. \tag{21b}$$

where we write $n_i = n_{z_i}$ to keep notation short.

Timing of the events is summarized in figure 3.

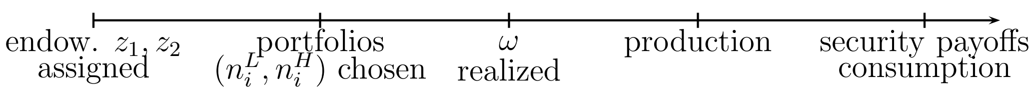

Figure 3: Timing of events

The first-order conditions for portfolio shares are:

$$\frac{U'(z + \alpha + 1 + n_z^H)}{U'(z + n_i^L)} = \frac{\pi^L}{1 - \pi^L} \cdot \frac{q^H}{q^L}, \quad z \in \{z_1, z_2\}. \tag{22}$$

The above implies that the ratio of marginal utilities is the same across individuals: This is a standard risk-sharing result that obtains here because markets are complete.

To understand portfolio decisions of the two types consider the case when the financial markets are closed. While a low-endowment type-1 individual has lower utility in every state his relative marginal value of consumption is higher in the low equilibrium:

$$\frac{U'(z_1)}{U'(z_2)} > \frac{U'(z_1 + \alpha + 1)}{U'(z_2 + \alpha + 1)}. \tag{23}$$

A sufficient condition for the above to hold is a decreasing absolute risk aversion that, in turn, is true if $U'''(c) > 0$.[3] So, we expect the low-endowment

[3]That is $-u''(c)/u'(c)$ must be decreasing.

type to purchase securities that pay in state $\omega = L$, $n_1^L \geqslant 0$, and sell securities that pay in state $\omega = H$ ($n_1^H \leqslant 0$). This intuition will be used to derive sufficient conditions for existence of equilibria.

2.3 Optimal portfolios with logarithmic utility

With $U(c) = \ln(c)$ equation (22) simplifies to:

$$\frac{z_1 + n_1^L}{z_1 + \alpha + 1 + n_1^H} = \frac{z_2 + n_2^L}{z_2 + \alpha + 1 + n_2^H} = \frac{\pi^L}{\pi^H}\frac{q^H}{q^L},$$

and implies:

$$\frac{q^H}{q^L} = \frac{\pi^H}{\pi^L}\frac{\bar{z}}{\bar{z} + \alpha + 1}.$$

Using the budget constraints and the market clearing conditions allows us solving for the optimal portfolios:[4]

$$n_2^L = -\pi^H f_1 \Delta_z \frac{\alpha + 1}{\bar{z} + \alpha + 1}, \quad n_1^L = \pi^H f_2 \Delta_z \frac{\alpha + 1}{\bar{z} + \alpha + 1}, \tag{24a}$$

$$n_2^H = \pi^L f_1 \Delta_z \frac{\alpha + 1}{\bar{z}}, \quad n_1^H = -\pi^L f_2 \Delta_z \frac{\alpha + 1}{\bar{z}}. \tag{24b}$$

with $\Delta_z = z_2 - z_1$. Notice that $n_2^H > 0$ as conjectured.

At the optimal portfolios agents achieve perfect insurance across the two equilibria. By this we mean that consumption of each type is a fixed, across the equilibria, fraction of the total good supply. This implies that consumption of any type in the L equilibrium is smaller than in the H equilibrium.[5] Then notice that $c_2^H > c_1^H$ because with the financial markets open the low-endowment type-1 repays the other type in the H equilibrium. Also in the L equilibrium consumption ordering is implied by the endowment ordering:

$$c_2^L - c_1^L = z_2 + n_2^L - z_1 - n_1^L = \Delta_z - \pi^H \Delta_z \frac{\alpha + 1}{\bar{z} + \alpha + 1} > 0.$$

[4] We use market clearing conditions to determine optimal purchases of securities by type-1 individuals: $n_1^\omega = -(f_2/f_1) n_2^\omega,\ \omega \in \{L, H\}$.

[5] This can be also proven directly. For type 1 we have $n_2^L > 0 > n_2^H$. Yet, because $f_2 \Delta_z < \bar{z}$, we get:

$$c_1^H - c_1^L = (\alpha + 1)\left[1 - \pi^L f_2 \Delta_z/\bar{z} - \pi^H f_2 \Delta_z/(\bar{z} + \alpha + 1)\right] \geqslant 0.$$

For type 2 the claim is trivial because $c_2^L = z_2 + n_2^L \leqslant z_2 < z_2 + 1 + \alpha \leqslant z_2 + 1 + \alpha + n_2^H = c_2^H$.

We state this result formally because we refer to it later.

Lemma 2. *$c_z^L < c_z^H, \forall z$ and there is no "consumption leapfrogging": $c_1^\omega < c_2^\omega, \omega = L, H$.*

Finally, we would like to point out the effect of group sizes. If each individual from a larger low-endowment type saved one unit then individuals in the other, smaller, group would receive more than one unit. For this reason, the payment to the high-endowment individuals in equilibrium H is rather large. But a large payment, as is shown later, may destroy equilibrium H. That is we expect the financial markets to have a strong effect on the set of possible equilibria when there is a sizable group of endowment-poor individuals. In societies with a small fraction of poor individuals opening the financial markets is unlikely to affect the set of equilibria. Yet, in the latter case significant improvement in risk-sharing across equilibria can be achieved. This is true because it costs little for the populous high-endowment group to insure a small group of poor. Formally, $|c_1^H - c_1^L|$ decreases as f_2 increases; see footnote 5. It is crucial to understand that low-endowment individuals demand insurance, and high-endowment individuals are willing to provide it, regardless of the group proportions (f_1, f_2). The size of the two groups matters for its effect on the financial market clearing – that is ability of one group to satisfy demands of the other.

2.4 Creating/destroying equilibria?

Suppose that without the financial markets only the low equilibrium exists. We now ask if it is possible that after the financial markets open both equilibria would exist. In the next section we ask if any of the equilibria could be destroyed.

Region L in figure 1: Suppose that when there are no financial markets only the L equilibrium exists: $z_2 \geqslant z_1 \geqslant \alpha\delta, z_2 \geqslant (\alpha+1)\delta$. When the financial markets are open the H and L equilibria exist if:

$$z_1 + n_1^L \geqslant \alpha\delta, \tag{25a}$$

$$z_2 + n_2^L \geqslant \alpha\delta, \tag{25b}$$

$$(\alpha + 1)\delta \geqslant z_1 + n_1^H, \tag{25c}$$

$$(\alpha + 1)\delta \geqslant z_2 + n_2^H. \tag{25d}$$

The first inequality always holds because $\alpha\delta \leqslant z_1, 0 \leqslant n_1^L$. The second inequality must be checked. The third inequality always holds because $z_1 \leqslant (\alpha+1)\delta, n_1^H \leqslant 0$. The fourth inequality cannot hold because $z_2 \geqslant (\alpha+1)\delta$ and $n_2^H \geqslant 0$. So, the H equilibrium cannot be created.

Region H in figure 1: Suppose that when there are no financial markets only the H equilibrium exists: $z_1 \leqslant z_2 \leqslant (\alpha+1)\delta, z_1 \leqslant \alpha\delta$. When the financial markets are open the H and L equilibria exist if the inequalities in (25) hold. The first and the second inequality could hold. But the third inequality cannot hold because $z_1 \leqslant (1+\alpha)\delta, n_1^H \leqslant 0$. So, the L equilibrium cannot be created either. We state these results in the following proposition.

Proposition 3. *Opening financial markets cannot create the H (L) equilibrium if only the L (H) equilibrium existed under financial autarky.*

We now ask if equilibria can be destroyed. Case 1(2) below studies if opening the financial markets can destroy the H (L) equilibrium if the two equilibria existed under financial autarky.

Region H+L in figure 1, case 1: Suppose that the H and L equilibria exist: $(\alpha+1)\delta \geqslant z_2 \geqslant z_1 \geqslant \alpha\delta$. When the financial markets are open only the H equilibrium exists if:

$$\alpha\delta \geqslant z_1 + n_1^L, \ \textit{or} \ \alpha\delta \geqslant z_2 + n_2^L, \tag{26a}$$

$$(\alpha+1)\delta \geqslant z_1 + n_1^H, \tag{26b}$$

$$(\alpha+1)\delta \geqslant z_2 + n_2^H. \tag{26c}$$

The third inequality in the above system cannot hold because $z_1 \leqslant (\alpha+1)\delta$ and $n_1^H \leqslant 0$.

Proposition 4. *Opening financial markets cannot destroy the L equilibrium if both equilibria existed under financial autarky.*

Region H+L in figure 1, case 2: Suppose that when there are no financial markets the H and L equilibria exist: $(\alpha+1)\delta \geqslant z_2 \geqslant z_1 \geqslant \alpha\delta$. When the financial markets are open only the L equilibrium exists if:

$$z_1 + n_1^L \geqslant \alpha\delta, \tag{27a}$$

$$z_2 + n_2^L \geqslant \alpha\delta, \tag{27b}$$

$$(\alpha+1)\delta \leqslant z_1 + n_1^H, \ \textit{or} \ (\alpha+1)\delta \leqslant z_2 + n_2^H. \tag{27c}$$

The first inequality always holds. The inequality $(\alpha+1)\delta \leqslant z_1 + n_1^H$ in the third row cannot hold. So, we need to check if the intersection of $\{z_2 + n_2^L \geqslant \alpha\delta, (\alpha+1)\delta \leqslant z_2 + n_2^H\}$ and $\{(\alpha+1)\delta \geqslant z_2 \geqslant z_1 \geqslant \alpha\delta\}$ is non-empty. This can be easily verified by setting $z_1 < z_2 = (\alpha+1)\delta$. In this case $n_2^H > 0$ and $z_2 + n_2^H > (1+\alpha)\delta$.

Proposition 5. *There exists a non-empty set of parameters such that opening financial markets can destroy the H equilibrium if both equilibria existed under financial autarky.*

2.5 Restricting equilibrium values of π^L

Suppose that when the financial markets are closed the L and the H equilibria exist: $(1+\alpha)\delta \geqslant z_2 \geqslant z_1 \geqslant \alpha\delta$. When the financial markets are open the H and L equilibria exist if:

$$z_1 + n_1^L \geqslant \alpha\delta, \tag{28a}$$
$$z_2 + n_2^L \geqslant \alpha\delta, \tag{28b}$$
$$(\alpha+1)\delta \geqslant z_1 + n_1^H, \tag{28c}$$
$$(\alpha+1)\delta \geqslant z_2 + n_2^H. \tag{28d}$$

The first inequality always holds because $z_1 \geqslant \alpha\delta$ and $n_1^L \geqslant 0$. The second inequality always holds because $|n_2^L| < \Delta_z$ and $z_1 \geqslant \alpha\delta$. The third inequality always holds because $(\alpha+1)\delta \geqslant z_1$ and $n_1^H \leqslant 0$. The fourth inequality must be verified. So, both equilibria survive if:[6]

$$(\alpha+1)\delta \geqslant z_2 + n_2^H \geqslant z_2 \geqslant z_1 \geqslant \alpha\delta. \tag{29}$$

After substituting the formula for n_2^H we obtain:

$$\pi^L \leqslant \frac{(\alpha+1)\delta - z_2}{(\alpha+1)f_1\Delta_z/\bar{z}} \equiv \bar{\pi}^L \tag{30}$$

The region where both equilibria exist before and after the financial markets open is plotted in figure 1, panel B. At the upper boundary of the union of the H and the H+L regions, the endowment-rich type 2 is indifferent between working and not.

[6]Notice that the inequality $(\alpha+1)\delta \geqslant z_2$ is redundant. This means that the set of parameters for which the H equilibrium exists shrinks when the financial markets open.

Together with the condition for the existence of the two equilibria under financial autarky, $\alpha\delta \leqslant z_1 \leqslant z_2 \leqslant (\alpha + 1)\delta$, inequality (30) is the restriction on equilibrium beliefs and model parameters under which the two equilibria exist regardless of the financial regime. Intuitively, the probability of the L equilibrium, π^L, cannot be too high as then the high-endowment type-2 individuals would not work in the high equilibrium and the latter would cease to exist. This happens because as π^L grows the relative price q^L/q^H and n_2^H increase. But when a payoff in any state increases incentives to work decrease. The restriction on π^L could also be vacuous, e.g. when $\Delta_z = 0$, or it could be "prohibitive," e.g. when $z_2 = (\alpha + 1)\delta$.

As explained above, the upper bound on π^L stems from the restriction that the high-endowment type-2 agents should support the H equilibrium. The term $(\alpha + 1)\delta - z_2$ is the largest trade that does not destroy type-2's incentives to work. The term $(\alpha+1)f_1\Delta_z/\bar{z}$ determines the size of the trade, see (24b). If there were no heterogeneity, $\Delta_z/\bar{z}$ is close to zero, then there would be no trade; so, any π^L would do. $(\alpha + 1)f_1$ is the additional income earned by the poor when the H equilibrium is selected. The larger it is the stronger are trading motives and, hence, higher chances of destroying the equilibrium. Figure 4 illustrates the relation between $\bar{\pi}^L$ and (α, δ). Notice that as α and/or δ increase the L equilibrium disappears. Similarly, when α and/or δ decrease the H equilibrium disappears. For intermediate value of (α, δ) the figure plots the limit on the probability of the L equilibrium. When α and/or δ are high, but not enough to destroy the L equilibrium, the probability of the L equilibrium is unrestricted. In this case the high-endowment type-2 individuals have a substantial "insurance capacity" and provide for the low-endowment individuals while continuing to work. This area corresponds to the plateau in the figure.

Observe that the upper bound on π^L is linear in δ and hyperbolic in α:

$$\bar{\pi}^L = [f_1\Delta_z/\bar{z}]^{-1}\left[\delta - \frac{z_2}{\alpha + 1}\right]. \tag{31}$$

It increases with δ as this expands the area where both equilibria are possible. As α increases, two effects are operational. First, it is harder to destroy the H equilibrium: the upper bound on consumption of a type-2 individual increases. Second, trades increase as they are proportional to $(1 + \alpha)$ measuring the increase in the aggregate consumption between the L and the H equilibrium. However, financial payoffs of any individual cannot not exceed $(1 + \alpha)$, and the first effect dominates.

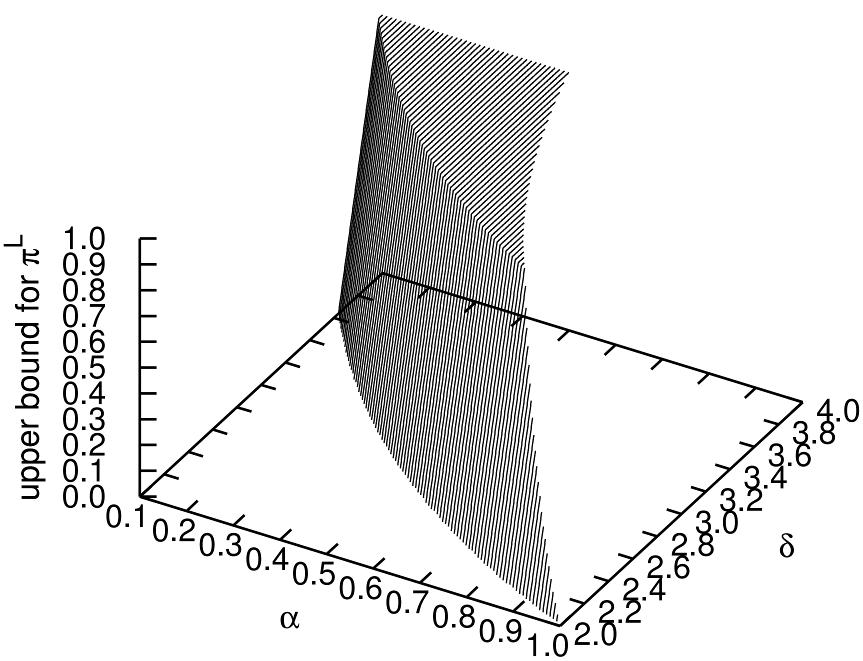

Figure 4: Relation between $\bar{\pi}^L$ and (α, δ).

Lastly, the upper bound on π^L depends on δ. This parameter has no effect on the size of financial trades or equilibrium prices. It also difficult to calibrate. For these reasons, we provide an alternative upper bound that does not involve δ. To this end, note that for equilibrium H to exist we must have $z_1 \geqslant \alpha\delta$. This imposes an upper bound on δ that can, in turn, be used in (31):

$$\bar{\pi}^L \leqslant [f_1 \Delta_z / \bar{z}]^{-1} \left[\frac{z_1}{\alpha} - \frac{z_2}{\alpha + 1} \right]. \tag{32}$$

Size of disasters vs. their frequency.—The size of disasters is governed by α – The larger is α, the more severe is the drop in the aggregate consumption – see eq. (2). If α is taken as a measure of disaster size, then the size and frequency of disasters are positively related: The larger the disaster, the higher is the likelihood that it can occur in equilibrium. Of course, this pertains only to coordination failures; the opposite is probably true of wars and natural catastrophes.

2.6 Dispersion of endowments

Rising inequality, as measured by $\Delta_z/\bar{z}$, reduces the probability of equilibrium L. The more dispersed endowments are the larger are incentives to trade in equilibrium for then the rich value consumption much less than the poor. On the other extreme, when endowments are similar there is little incentives to trade. In this case the set of possible sunspot equilibria is unaffected as $\bar{\pi}^L \geqslant 1$ is not restrictive. When dispersion is small, $\Delta_z/\bar{z} \leqslant [\delta - z_2/(\alpha+1)]/f_1$ according to (31), then opening the financial markets has no effect on the probability of equilibrium L. This implies that if a fictitious planner could redistribute endowments across individuals he would not choose an equal distribution. That is increased inequality has a positive welfare effect.

2.7 The set of equilibria, $\mathcal{A}$

Having established a perfect correlation between asset positions and actions, we may abbreviate the definition of equilibrium as follows: Instead of the objects defined in (14) and (15), we shall refer to equilibrium as the set $A = \mathbf{L}$ of ω values for which agents all set $x = 0$. I.e., it is the set of ω's for which equilibrium L results. The gross asset positions $N(\cdot)$ of the two types of agents then follow straightforwardly.

The equilibrium set $\mathcal{A}$.—The equilibrium is any set of disaster states the measure of which does not exceed $\bar{\pi}^{\mathrm{L}}$. I.e., is the collection of Borel subsets $A \subset [0,1]$ for which $\pi^A \leq \bar{\pi}^{\mathrm{L}}$. Thus the set of equilibria is the set

$$\mathcal{A} = \left\{ A \in \mathcal{B}([0,1]) \mid \int_A d\mu(s) \leq \bar{\pi}^{\mathrm{L}} \right\}. \tag{33}$$

We have provided only an upper bound on π^L. One may ask whether the use of asset trades can narrow things down further if the game were different in some way. We can see two options for narrowing down the set equilibrium π^L. One way is to use the theory of the Core in which competition occurs among coalitions, i.e., a theory in which groups of agents can deviate from any outcome. A second way to reduce the number of equilibria is to add stages to the security trading game. Banks could propose securities by sending messages to agents who then would choose where to trade. Using the

Core equilibrium concept would lead to an open set problem in the coalitions' choice of π^L for the following reason: The upcoming Lemma shows that a smaller value of π^L Pareto dominates a larger, recognizing, of course, that the equilibrium asset prices q^{L} and q^{H} depend on π^L. In other words, the equilibria, as indexed by π^L, are Pareto ranked. This is our next result.

2.7.1 All agents are better off in equilibrium H

The utility of a type-i individual is: $W_i = \pi^{\mathrm{L}} U(z_i + n_i^L) + (1-\pi^L) U(z_i + \alpha + 1 + n_i^H)$. We will later see that the type-1's portfolio positions (n_1^L, n_1^H) decrease with π^L. Hence, utility of a type-1 individual is strictly decreasing in π^L. The type-2's portfolio positions, on the other hand, increase with π^L. That is, as the probability of L rises, consumption of a type-2 individual increases in both states but his overall utility still falls as H becomes less likely. Lemma 6 shows that W_2 is decreasing in π^{L} as long as $\pi^L \pi^H f_1 \Delta_z / \bar{z} < 0.5$. This constraint is not vacuous. But it is also not restrictive as it would be satisfied if, for example, $\Delta_z < 2\bar{z}$.

Lemma 6. *If* $\pi^L \pi^H f_1 \Delta_z / \bar{z} < 0.5$ *then* $dW_i/d\pi^L < 0, i = 1, 2$.

Given this, competition among coalitions would lead them towards the Pareto-optimal outcome. But at $\pi^L = 0$ there can be no trade. We then would be back in a no-financial-asset game that admits both equilibria, L and H.

Alternatively, we may add a prior stage to the security trading game. Banks could propose securities by sending messages to agents who then would choose where to trade. It appears that this could be formulated so as to lead to the same outcome as the Core with the same open set problem. At the moment, then, we cannot shrink $\mathcal{A}$ any further.

2.8 Asset pricing

Suppose now that individuals also receive endowment z_0 in period 0 before types are revealed in period 1. Type-i individual receives endowment z_i and chooses whether to work or not as before. In period 0 individuals are offered to buy (equity) claims to the aggregate output Y^ω,

$$Y^\omega \equiv \begin{cases} \alpha + 1 & if \omega = H, \; prob = 1 - \pi^L \\ 0 & if \omega = L, \; prob = \pi^L \end{cases} . \tag{34}$$

and the risk-free bond that pays one unit of consumption regardless of the realized ω. The two assets are traded at prices q^e and q^b that will be determined later. Timing of events is as follows:
1. Trade risk-free bonds and claims to the aggregate output, consume;
2. Learn your z, trade state-contingent portfolios, produce and consume.

The period 0 budget constraint is:

$$c_0 + q^e n_0^e + q^b n_0^b = z_0. \tag{35}$$

Since all individuals are symmetric in period 0 we do not use index i. For the same reason purchases of the two assets, equity claim and bond, is zero in equilibrium:

$$n_0^e = n_0^b = 0. \tag{36}$$

So, everyone simply consumes his endowment: $c_0 = z_0$. The two asset prices satisfy the following Euler equations:

$$q^b = \beta E\left[\frac{U'(z_i + n_i^\omega)}{U'(z_0)} 1\right], \tag{37a}$$

$$q^e = \beta E\left[\frac{U'(z_i + n_i^\omega)}{U'(z_0)} Y^\omega\right]. \tag{37b}$$

The returns on the two assets then are:

$$R^b = 1/q^b, \quad R^e = E[Y^\omega]/q^e. \tag{38}$$

The interim expected utility:

$$V_z(\pi^L, n^b, n^e) = \pi^L U(z+n^b+n_z^L)+(1-\pi^L)[U(z+\alpha+1+n_z^H+n^b+(\alpha+1)n^e)-\kappa].$$

The life-time utility

$$\max_{n^b, n^e} U(z_0 - q^b n^b - q^e n^e) + \beta \sum_{z\in\{z_1,z_2\}} f_z V_z(\pi^L, n^b, n^e).$$

Price of the risk-free bond at $n^b = n^e = 0$ is:

$$q^b = \beta \sum_z f_z \frac{\pi^L U'(z + n_z^L) + (1 - \pi^L) U'(z + \alpha + 1 + n_z^H)}{U'(z_0)}. \tag{39}$$

Price of a claim to the aggregate endowment (equity) is:

$$q^e = \beta \sum_z f_z \frac{U'(z + \alpha + 1 + n_z^H)}{U'(z_0)} (1 - \pi^L)(\alpha + 1). \tag{40}$$

Then the expected return on equity is:

$$E[R^e] = \frac{(\alpha + 1)(1 - \pi^L)}{q^e} = \frac{U'(z_0)}{\beta \sum_z f_z U'(z + \alpha + 1 + n_z^H)},$$

where the optimal portfolios are:

$$n_1^H = -\pi^{\mathrm{L}} f_2 \Delta_z \frac{\alpha + 1}{\bar{z}}, \quad n_2^H = \pi^{\mathrm{L}} f_1 \Delta_z \frac{\alpha + 1}{\bar{z}}.$$

As the probability π^L increases, probability that an equity claim pays decreases. So, the equity claim is valued less and it must offer a higher return. At the borderline case with $\pi^L = 0$ the risk-free bond and the equity claim yield the same return. We state these results in the following proposition.

Proposition 7. *With logarithmic preferences the expected equity premium is a) always non-negative, and b) an increasing function of π^L.*

Proof. By direct differentiation. □

Next, we compute the price of a disaster insurance. The disaster insurance pays one unit of consumption good when the L equilibrium realizes. Notice that the risk-free bond pays $(1, 1)$ in the two states and a claim to equity pays $(\alpha + 1, 0)$. Then a disaster insurance claim generates the same payoff as a portfolio comprised of 1 bond and $-\frac{1}{\alpha+1}$ equity claims. So, in the absence of arbitrage the price of the disaster insurance must be:

$$q^d = q^b - \frac{1}{\alpha + 1} q^e = \beta \sum_z f_z \frac{\pi^L U'(z + n_z^L)}{U'(z_0)}. \tag{41}$$

Infinite horizon: for better quantitative results we could study a repeated game.

2.9 News shock

The simplest treatment of a news shock is a prior signal ξ on s, drawn from the density $g\left(\xi \mid s\right)$. Denote the posterior over s by $\mu\left(s \mid \xi\right)$. This in general makes the states not equally likely but the main thing is that the signal changes the disaster probability from $\mu\left(A\right)$ to $\mu\left(A \mid \xi\right)$.

In order that the previous analysis should apply, however, it is easier to have the new shock leave the likelihood of s unchanged, but to change the designation of which equilibrium is associated with which value of s. We now put a prior distribution ν over $\mathcal{A}$ and, derived from ν, a prior distribution λ over $\left[0, \bar{\pi}^{\mathrm{L}}\right]$. The news consists of an announcement of a particular $A \in \mathcal{A}$ and, hence, an implied value for $\pi^L \in \left[0, \bar{\pi}^{\mathrm{L}}\right]$. The measure ν is an object different from μ; the latter tells us the likelihood of various ω's occurring, whereas ν tells us the likelihood of which combinations of the ω's are to lead to equilibrium L. Thus the measure ν generally will not be Lebesgue measure μ but, rather, can put greater weight on some Borel subsets of $\mathcal{A}$ and less weight on others.

In other words, a news shock is an announcement of the list of $\omega \in [0, 1]$ that are to be considered disaster states. If many ω's are announced to be disaster states, then disasters become more likely, and this will affect asset prices as well as asset trading. The list of disaster states will be denoted by A. Suppose that the announced A is drawn randomly from the equilibrium set $\mathcal{A}$ taking $\nu\left(A\right)$ as the measure. This implies π^L which is drawn randomly from the set of numbers not exceeding $\bar{\pi}^{\mathrm{L}}$. The prior measure over π^L is λ, where

$$\lambda(\pi^L) = \int_{\mathcal{A}} \mu\left(A\right) d\nu\left(A\right) \tag{42}$$

When A is announced, beliefs shift from ν to a point mass on A or, from λ to a point mass on π^L. This has the interpretation of a belief shock, since it does not affect fundamentals. From now on we shall refer to the news shock as the revelation of a specific value $\pi^L \in \left[0, \bar{\pi}^{\mathrm{L}}\right]$.

Do stock prices lead output?—We ask if q^e is a leading indicator of the aggregate output Y^ω. Conditional on π^L, expected output is $E[Y] = (1 -$

$\pi^L)(\alpha+1)$. Then before π^L is revealed asset prices are:

$$\tilde{q}^b = \beta \int_0^{\bar{\pi}^L} \frac{\sum_z f_z[\pi^L U'(z+n_z^L) + (1-\pi^L)U'(z+\alpha+1+n_z^H)]}{U'(z_0)} d\lambda(\pi^L), \tag{43a}$$

$$\tilde{q}^e = \beta \int_0^{\bar{\pi}^L} \frac{\sum_z f_z U'(z+\alpha+1+n_z^H)}{U'(z_0)} (\alpha+1)(1-\pi^L) d\lambda(\pi^L). \tag{43b}$$

The news effect is the difference between the expected price of a portfolio and the realized price after the π^L is revealed:

$$New\omega se \equiv \tilde{q}^e - \beta \sum_z f_z \frac{U'(z+\alpha+1+n_z^H)}{U'(z_0)} (1-\pi^{\mathrm{L}})(\alpha+1), \tag{44a}$$

$$New\omega sb \equiv \tilde{q}^b - \beta \sum_z f_z \frac{\pi^{\mathrm{L}} U'(z+n_z^L) + (1-\pi^L)U'(z+\alpha+1+n_z^H)}{U'(z_0)}. \tag{44b}$$

Because price of equity is a decreasing function of π^L it is positively correlated with the expected aggregate output $E[Y]$. So, the stock market index is a leading indicator of output.

The financial market volume[7] is:

$$v = \sum_{w \in \{H,L\}} |f_2 n_2^w| = (1+\alpha) f_1 f_2 \Big\{ \frac{\pi^{\mathrm{L}}}{\bar{z}} + \frac{\pi^H}{\bar{z}+\alpha+1} \Big\}. \tag{45}$$

So, when π^L increases the market volume also increases. That is, the trading volume leads the aggregate output.

In a related paper, Angeletos and La'O (2014) also study shocks to beliefs about the actions of others. They do not have multiplicity of equilibria as we do, but they instead have aggregate shocks. The presence of the latter, they show, also allows shocks to beliefs over actions to have real effects.

Is lagged consumption a sufficient statistic for current consumption?—Hall (1978) derived the implication that no variable apart from current consumption should be of any help in predicting future consumption. Hall did find that real disposable income did not help predict aggregate consumption, but that an index of stock prices did help predict it. In our two-period model

[7] A symmetric formula can be defined using positions of a type-1 individual.

the question can be posed as follows: Is z_0 a sufficient statistic for predicting y? The answer is "no" since news to π^{L} cannot be reflected in z_0 which is an endowment, and yet low π^L is a good news for Y and, hence, for the consumption of all agents. Although the proportions consumed by each type do change with π^L, lemma 2 shows that the consumption of each type is higher in equilibrium H than in equilibrium L.

A low realization of π^L is also a good news for the equity price, indicating that equity prices can help predict future consumption. Assume that z_0 is a random variable drawn from a known distribution. News then consists of a simultaneous "announcement" of (z_0, π^L) that is then followed by trade in the financial markets. It turns out that stock price is also not a sufficient statistic for Y. The level of prices depends on z_0 and therefore one needs to know z_0 in order to be able to predict future consumption. But knowledge of the pair (z_0, q^e) is sufficient to predict future consumption, consistent with what Hall finds empirically. Formally, consider a first-order approximation of q^e around $(z_0, \pi^L) = (E(z_0), 0) : q^e = k_0 + k_z z_0 - k_\pi \pi^L$ where $k_0, k_z, k_\pi > 0$. Expected aggregate consumption is: $E(C^w) = \bar{z} + \alpha + 1 - \pi^L(\alpha + 1)$. Then consider the following regression specification relating the expected consumption to the first-stage aggregate consumption z_0 and the equity price q^e: $E(C^w) = \beta^0 + \beta_z z_0 + \beta_q q^e = \beta^0 + \beta_z z_0 + \beta_q(k_0 + k_z z_0 - k_\pi \pi^L)$. One should find significant β_z and β_q. Moreover, β_q should be positive while the coefficient β_z should be negative.[8]

2.10 Illustrative example

Heathcote, Storesletten, and Violante (2006, Figure 4) report that an average of the variance of log wages and the variance of log earnings for a 33-year-old worker is 0.33. That is

$$var(z) = f_1 f_2 (ln(z_2) - ln(z_1))^2 = 0.33. \tag{46}$$

With $f_1 = 0.50$ we get $z_2/z_1 = x \equiv exp(2/\sqrt{3}) \approx 3.17$. So, we get: $z_1 = \bar{z}2/(1+x), z_2 = \bar{z}2x/(1+x)$. The restrictions imposed by existence of both equilibria are: $\alpha\delta \leqslant z_1 \leqslant z_2 \leqslant (\alpha+1)\delta$.

We assume $\delta = 3.5$. We choose $\bar{z} = 2.82, \alpha = 0.14$ so that $\bar{z}/(\bar{z}+\alpha+1) \approx 0.71$ as in Barro (2006) and the implied upper bound on π^L is 0.020, similar to Barro's (2006) estimate of 0.017.

[8] Simple coefficient matching gives: $\beta_q = (\alpha+1)/k_\pi > 0, \beta_z = -\beta_q k_0 < 0$.

We set z_0 so that no growth is expected in the aggregate consumption:

$$z_0 = \bar{z} + (\alpha + 1)E[\pi^L]. \tag{47}$$

variable	value	moment
δ	3.50	–
f_1	0.50	Groups of equal size
(z_1, z_2)	(1.32,4.28)	Coefficient of variation for endowment is 0.33
α	0.23	29% loss of output in the L equilibrium
z_0	see (47)	Expected consumption growth is zero

Table 1: Parameters for the numerical example

Table 1 collects all the parameter assumptions. Figure 5 plots returns of the risk-free bond, the equity and the disaster claims. It is assumed that $z_0 = \bar{z} + (a + 1)(1 - \pi^L)$, that is the expected aggregate consumption is constant. The vertical line marks the upper bound on the probability of equilibrium L, $\bar{\pi}^L$. When $\pi^L = 0$ then there is only one state of the world – the H equilibrium – and the equity claim and the bond pay the same. When π^L reaches its upper bound 0.02 the return on equity is 0.64% and the risk-free return is -0.23% implying a premium of 0.87%. Despite being relatively small, the premium in the data is about 5%, we would like to emphasize that this premium reflects only the endogenous disaster risk as there are no other sources of uncertainty in the model. As another comparison consider the results in Barro (2006): assuming logarithmic preferences this model predicts only 0.24% premium.[9] The premium and the return on the disaster claim are increasing in π^L. At $\pi^L = 0.04$ the premium is sizeable and measures 1.74%. Finally, the return on equity and the risk-free bond are much higher than that if the disaster claim. The reason for this is that individuals expect a higher consumption growth if equilibrium H realizes. This makes the disaster claim to be very attractive as it pays when consumption is scarce; so, individuals would be willing to purchase it despite the low return that it offers.

[9] We assume that the bond is risk-free, that is it pays fully even if a disaster occurs. Then, assuming logarithmic preferences, the premium equals approximately $\sigma_c^2 + \pi^d(1 - d)(1/d - 1)$ where σ_c is the consumption growth volatility, π^d is the probability of a disaster, d is the output 'saved' in a disaster state. Setting $\sigma_c = 0$ we are left with the premium component that stems from the disaster risk alone. Setting $\pi^d = \bar{\pi}^L = 0.02$ and $d = 0.71$ we get $\pi^d(1 - d)(1/d - 1) = 0.0024$.

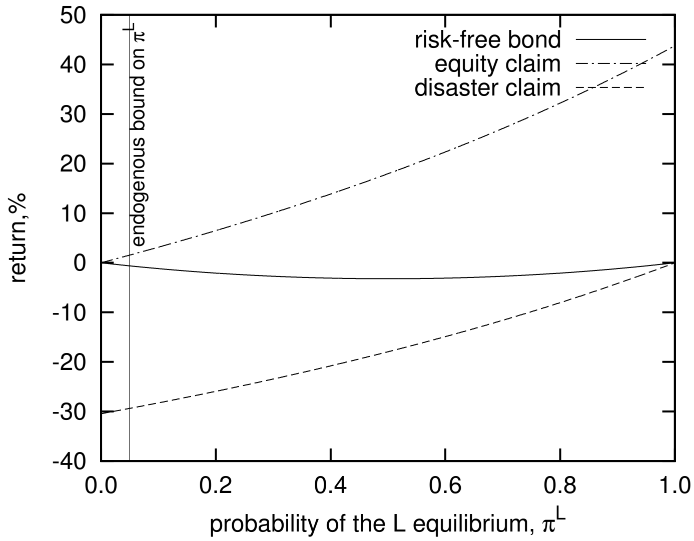

Figure 5: Return on the bond and the claim to the aggregate output.

3 Extensions of the results

We now discuss how the model relates to several lines of research.

3.1 Correlated equilibria

The title of the paper notwithstanding, our methods carry over to the case in which the sunspot is not observable, i.e., when the coordinating device is not contractible or securitizeable. In a correlated equilibrium in which agent i receives a private message η_i, and the η's are correlated. And yet we will now show that in principle asset trading will generally place restrictions on the correlated equilibria in much the same way as it places restrictions on the sunspot equilibria.

Since it is not reasonable to have markets on individual signals, it is reasonable to ask if our results carry over to correlated equilibrium without an aggregate public signal. To discuss this we must distinguish assets such as Arrow securities that pay as a function of the realization of some exogenous *state*, from assets that pay as a function of the set of *actions* that agents take. An example of the latter is an index fund linked to the S&P 500 that pays based on earnings that, in turn, depend on actions taken by the economy's agents.

A correlated equilibrium is a bivariate distribution over actions x_i and messages η_i that players receive. In this anonymous game in which no player can influence the aggregate outcome, the distribution of the signals alone then determines the distribution of actions. Thus in a correlated equilibrium the distribution of actions is determined by the empirical distribution of the signals. In the limit as the number of agents gets large, the empirical distribution of the signals coincides with the theoretical distribution. Well-known representation results allow us to relate this to an aggregate extrinsic statistic such as the scalar $s \in [0,1]$. We can index equilibrium play by the empirical distribution of the signals and as long as no individual agent can influence the aggregate outcome, asset trading based on the distribution of actions (when they are securitizeable) is equivalent to trading on s.

De Finetti's (1931) representation theorem implies that our results carry over exactly when the signals are binary, as the economy gets large. Suppose that agents are equally likely to get a signal so that the signals are exchangeable. De Finetti's result states that the $(\eta_i)_{i=1}^{\mathrm{N}}$ is a sequence of exchangeable Bernoulli random variables if and only if the CDF of the vector η has the representation

$$\Pr(\eta_1, ..., \eta_{\mathrm{N}}) = \int s^{t_{\mathrm{N}}} (1-s)^{\mathrm{N}\text{-}t_{\mathrm{N}}} \, d\mu(s) \tag{48}$$

where μ is a measure on $[0,1]$, though not necessarily Lebesgue measure as we have assumed. Let $\nu_N\left(\eta^{\mathrm{N}}\right) \equiv \Pr(\eta_1, ..., \eta_{\mathrm{N}})$. For fixed N there is a one-to-one correspondence between ν_{N} and s. For finite N, the empirical distribution will generally differ from ν_{N} but as $N \to \infty$, they coincide almost surely because our game has no discontinuities at $N = \infty$. And if equilibrium play depends on ν_{N} and not on players' names as $N \to \infty$, s becomes a sufficient statistic for all the moments ν_{N} and, hence, for what equilibrium gets selected. In other words, trading on "sunspots" cannot improve payoffs over trading on the equilibrium distribution of actions.

When the signal space is not binary, the representation results do not generally allow the correlated equilibrium to be representable by a sunspot belong to the unit interval. Here we may cite the extension of de Finetti's results by Hewitt and Savage (1955). When, for example the $\eta_i \in R$ are exchangeable, instead of (48) we have

$$\nu_{\mathrm{N}}\left(\eta^{\mathrm{N}}\right)=\int_{\Delta(R)}\prod_{i=1}^{\mathrm{N}}s\left(\eta_i\right)d\mu\left(s\right)$$

where μ is no longer a measure on the unit interval, but a measure over $s\in\Delta(R)$, i.e., a measure over measures on the line. The dimensionality of what one may call a sunspot is then generally larger than a scalar variable as we have modeled it.

3.2 Global games

Our results also apply to "global games" in which there are intrinsic (payoff relevant) aggregate shocks. Instead of writing the output equation as $y=(\alpha+\bar{x})x$, we may alternatively write it as:

$$y=(1+\alpha\bar{x})x,$$

so that α could represent the return to a currency attack or some other coordination game. Then we could assume that $\alpha\in\{0,1\}$ and agents do not know the realization of α. It is known that in such situations a little uncertainty can, under certain informational assumptions, lead to a unique equilibrium. This is a different way of restricting the set of equilibria in games that involve intrinsic uncertainty, as Goldstein and Pauzner (2005) have shown in the context of bank-run models. Our model restriction on equilibria applies to such models too, at least when the uncertainty over α is large enough so that the Carlsson and Van Damme (1993) argument cannot eliminate the multiplicity.

Conclusion

In a model in which multiple Pareto-ranked equilibria may arise, we have distinguished between sunspots and the equilibria that result therefrom. By introducing asset trading we have endogenized the mapping from the sunspot to equilibrium play and derived a bound on the probability with which the disaster equilibrium occurs.

We have then used the model to analyze several phenomena, including the effects of shocks to beliefs about the actions of others and how they manifest themselves in asset prices, and the relation between disaster size and probability on the one hand, and the disaster premium on the other.

Finally, we have shown that asset trading can reduce the incidence of coordination failures. Our model points to costs and benefits stemming from changes in the equilibrium set.

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A Proof of lemma 6

Proof.

$$\begin{aligned}\frac{dW_q}{d\pi^L} =& \underbrace{u(z_1 + n_1^L) - u(z_1 + \alpha + 1 + n_1^H)}_{negative} \\ &+ \underbrace{\pi^L u'(z_1 + n_1^L)\frac{dn_1^L}{d\pi^L} + (1-\pi^L)u'(z_1 + \alpha + 1 + n_1^H)\frac{dn_1^H}{d\pi^L}}_{both\ terms\ are\ negative} < 0.\end{aligned}$$

Next

$$\begin{aligned}\frac{dW_2}{d\pi^L} =& \underbrace{u(z_2 + n_2^L) - u(z_2 + \alpha + 1 + n_2^H)}_{negative} \\ &+ \underbrace{\pi^L u'(z_2 + n_2^L)\frac{dn_2^L}{d\pi^L} + (1-\pi^L)u'(z_2 + \alpha + 1 + n_2^H)\frac{dn_2^H}{d\pi^L}}_{both\ terms\ are\ positive} \\ =& u(z_2 + n_2^L) - u(z_2 + \alpha + 1 + n_2^H) \\ &+ \pi^L \pi^H f_1 \Delta_z (\alpha + 1)\left[\frac{u'(z_2 + n_2^L)}{\bar{z} + \alpha + 1} + \frac{u'(z_2 + \alpha + 1 + n_2^H)}{\bar{z}}\right],\end{aligned}$$

where the last equality relies on the optimal portfolios derived in 24b. Then by the concavity of u and the fact that $u'(z_2+\alpha+1+n_2^H)/u'(z_2+n_2^L) = (\bar{z}+\alpha+1)/\bar{z}$ we get

$$\begin{aligned}
\frac{dW_2}{d\pi^L} \leqslant & -u'(z_2+a+1+n_2^H)(\alpha+1) \\
& + \pi^L\pi^H f_1\Delta_z(\alpha+1)\left[\frac{u'(z_2+n_2^L)}{\bar{z}+\alpha+1}+\frac{u'(z_2+\alpha+1+n_2^H)}{\bar{z}}\right] \\
= & -u'(z_2+a+1+n_2^H)(\alpha+1)+2\pi^L\pi^H f_1\Delta_z(\alpha+1)u'(z_2+\alpha+1+n_2^H)/\bar{z} \\
= & u'(z_2+a+1+n_2^H)(\alpha+1)[-1+2\pi^L\pi^H f_1\Delta_z/\bar{z}] < 0.
\end{aligned}$$

Notice that all of the derivations used the fact that $u(c) = ln(c)$. □

B Contour plot of $\bar{\pi}^L$

Figure 6 plots contours of $\frac{(\alpha+1)\delta-z_2}{(\alpha+1)f_1\Delta_z/\bar{z}} \equiv \bar{\pi}^L$ for the parameters described in 1. To have multiple equilibria with trading of assets we need $\delta \geq \frac{z_2}{1+\alpha}$. When this holds as an equality, we have $\bar{\pi}^{\mathrm{L}} = 0$, which is the $\bar{\pi}^{\mathrm{L}} = 0$ contour.

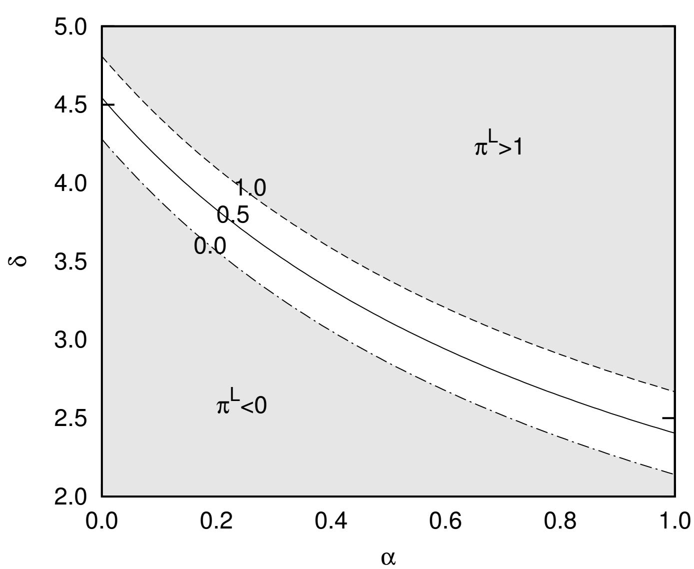

Figure 6: Contours of $\bar{\pi}^L$